Freezeout, fluctuations, fusion

Sandeep Chatterjee (VECC Kolkata)
and
Sourendu Gupta (TIFR Mumbai)

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1. Introduction
2. Examining reaction kinetics
3. Deuteron production
4. Summary

Outline

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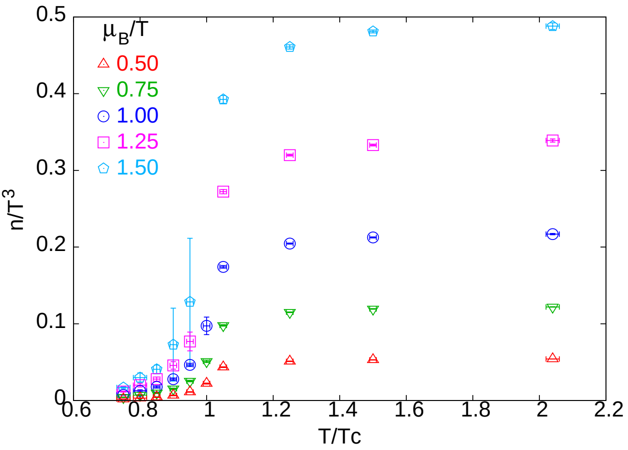
The EOS at finite density from the lattice



SG, Karthik, Majumdar 2014

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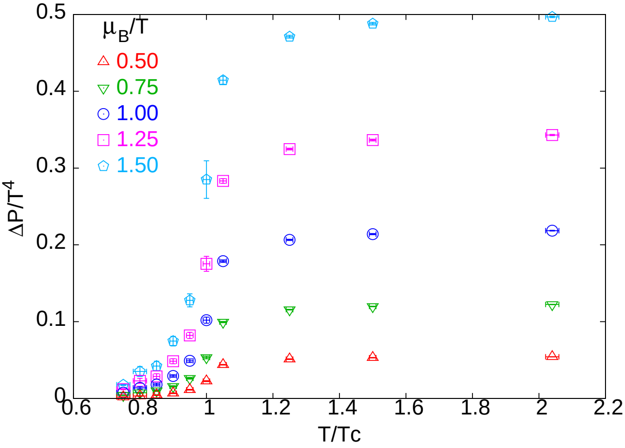

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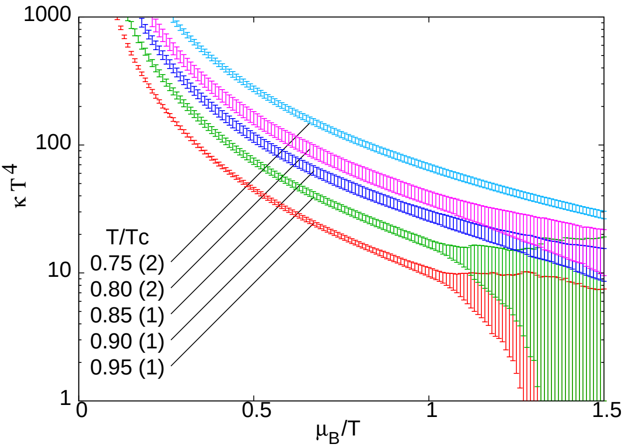

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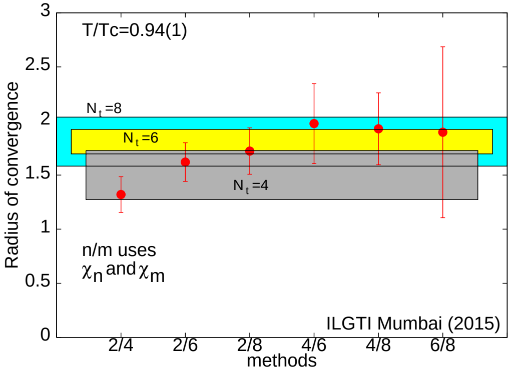
The critical end point from the lattice



Datta, Gavai, SG / SG, Karthik, Majumdar 2014

Is there thermal equilibrium? Can we understand fluctuations at freezeout. What is FO?

Big Bang Freezeout

Early universe hot and small: cooled and expanded. Particles froze out over wide range of T.

- **Cold Dark Matter** froze out at $m/T \simeq 30$; enough time after that to create structure in the universe through its gravitational interactions.
- **Neutrinos** froze out 0.2 s after BB, $T = 3$ MeV. Neutrons, muons could begin to decay.
- **Light nuclei** froze out in about 3 minutes, $T \simeq 0.1$ MeV: BBN puts strong constraints on the baryon/photon ratio in the universe, and therefore on Ω_b.
- **Photons** froze out when $T \simeq 0.3$ eV: realm of today's observational cosmology, fluctuation probes of the early universe.

Basics of freezeout

Boltzmann equation has two competing effects:

$$\frac{\partial f}{\partial t} = -\Gamma(f - f_{eq}) + v\frac{\partial f}{\partial r}$$

Microscopic physics of reaction rates: $\Gamma = n\sigma c_s$ where the number density $n = \int d^3 p f(p)$.

Global physics of expansion: v is flow velocity of a fluid. In cosmology more common to quote Hubble constant $H = v/r$.

When $H \ll \Gamma$ then reactions are (nearly) in equilibrium. When $H \simeq \Gamma$, reactions are driven out of thermal equilibrium.

When $H \gg \Gamma$ expansion freezes the final thermal fluctuations and expands it out to large scales.

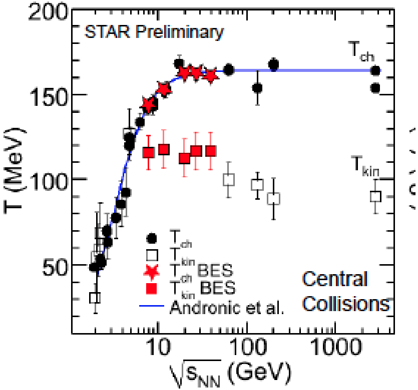
Little bang freezeout

Only strong interactions are (possibly) in equilibrium in heavy-ion collisions. Neutrinos, leptons, most detectable photons are not in equilibrium. Study of FO confined to hadrons.

1990s: the big success stories of LBF. Data confirmed that all hadrons freeze out nearly simultaneously: ideal gas of hadron resonances a good description of detected hadrons. Note: relic photons well described by Planck distribution does not mean the early universe was an ideal gas

2010s: more data, more precise data from RHIC-BES and LHC. Begin to look for more details. Try to understand the reaction kinetics. Begin to study the final departure from equilibrium: improve HRG, sharpen notion of chemical FO.

Hard but solvable problem



Estimates of chemical FO temperatures close to the QCD cross over or CEP: highly non-perturbative. Estimates of kinetic FO somewhat lower but with large errors: hadron dynamics important.

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When does isospin freeze out?

The rates for processes $p + \pi^- \leftrightarrow n + \pi^0$, remain high at 100 MeV, because $m_n - m_p$ is small and the yield of pions is large. So the CFO of isospin can be delayed.

Asakawa, Kitazawa, 2011

In the warm fireball, pion densities are $n_\pi \simeq (m_\pi T)^{3/2}/e$. The nucleon density is $n_N \simeq (m_N T)^{3/2} \exp(-m_N/T)$. The ratio is

$$\frac{n_N}{n_\pi} \simeq \left(\frac{m_N}{m_\pi}\right)^3 \exp\left(-\frac{m_N}{m_\pi} + 1\right) \simeq 0.06$$

So the nucleons exist in a isospin bath of pions.

The $p \leftrightarrow n$ reaction may proceed without suppression right up to KFO.

A toy model for isospin

$$\begin{aligned}
\dot{p} &= -\gamma(p\pi^0 - n\pi^+) - \gamma'(p\pi^- - n\pi^0) + \cdots\,, \\
\dot{n} &= \gamma(p\pi^0 - n\pi^+) + \gamma'(p\pi^- - n\pi^0) + \cdots\,, \\
\dot{\pi}^0 &= -\gamma(p\pi^0 - n\pi^+) + \gamma'(p\pi^- - n\pi^0) + \cdots\,, \\
\dot{\pi}^+ &= \gamma(p\pi^0 - n\pi^+) + \cdots\,, \\
\dot{\pi}^- &= -\gamma'(p\pi^- - n\pi^0) + \cdots\,.
\end{aligned}$$

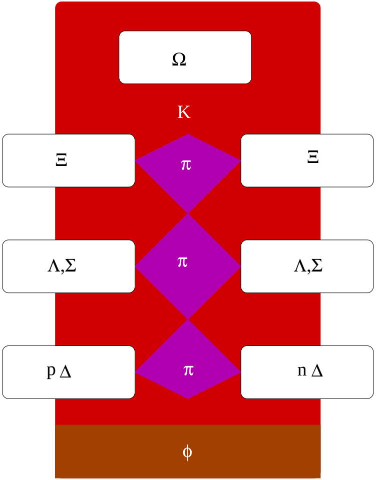
Rate constants γ and γ' from experiment. The equilibrium concentrations are given by

$$\frac{p}{n} = \frac{\pi^+}{\pi^0} = \frac{\pi^0}{\pi^-} \quad (= \zeta).$$

ζ is the isospin fugacity. Since $\pi^+/\pi^- = \zeta^2$, and observations are $\zeta \simeq 1$, then $\mu_I = T \log \zeta \simeq 0$.

No more observations! No motivation to improve.

Classic chemical freeze out



Parameters: T, μ, V, γ_S

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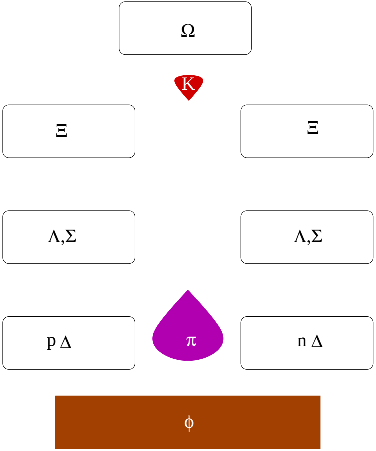


Parameters: T, μ, V, γ_S

Can the K and π freeze separately?

Indirect transmutations of K and π involve strange baryons in reactions such as $\Omega^- + K^+ \leftrightarrow \Xi^0 + \pi^0$. These have very high activation thresholds.

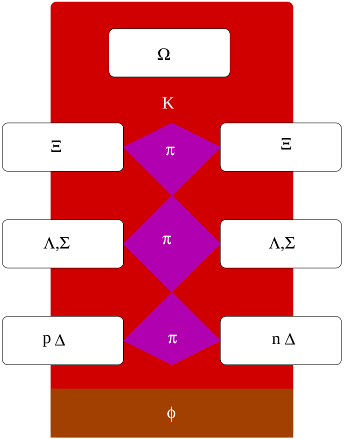
Direct transmutations can proceed through the strong interactions such as $K^+ + K^- \leftrightarrow \pi^+ + \pi^-$. These are OZI violating reactions; slower than generic strong-interaction cross sections.

Direct transmutations through weak interactions are not of relevance in the context of heavy-ion collisions.

There is no physics forcing K and π FO together. But K and ϕ are resonantly coupled, so FO together.

Chatterjee, Godbole, SG, 2013

Two-step chemical freeze out



Two extra parameters: T_S, μ_S, V_S, T_{NS}, μ_{NS}, V_{NS}

Chatterjee, Godbole, SG, 2013; Bugaev et al, 2013

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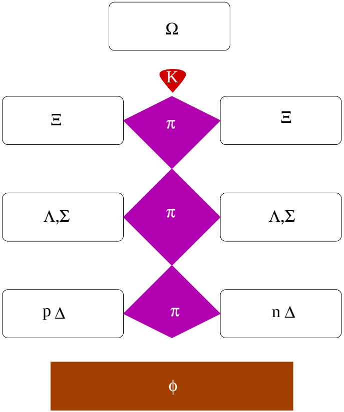

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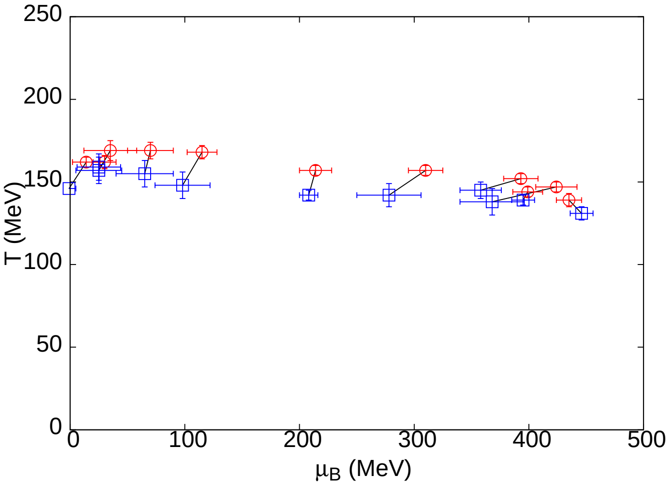
The freezeout curves



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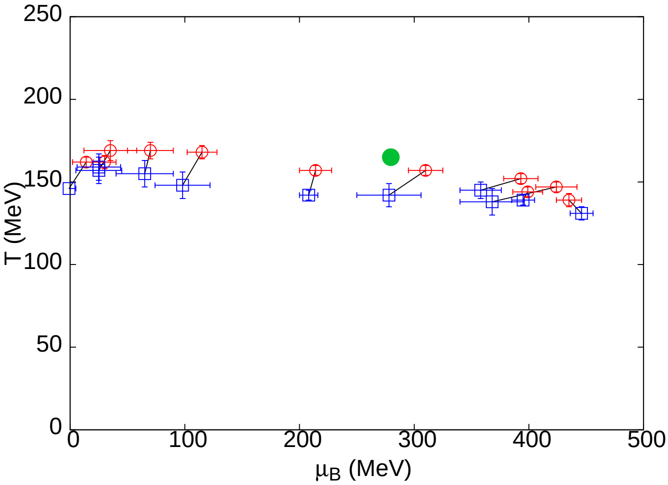

What about resonances?

Reaction kinetics of

$$K\pi \leftrightarrow \pi K^*, \qquad N\pi \leftrightarrow \pi N^*, \qquad \rho\pi \leftrightarrow \pi\pi.$$

When do these freeze out? No flavour quantum numbers changed by these reactions (except, possibly, isospin). Since $\pi \gg N$, N^* can be constantly regenerated. Similarly for ρ K^*?

Reaction kinetics easy to set up. Input parameters not well determined, energy dependence of cross sections may need to be modeled. More important: resonances decay by strong interactions, mostly not seen in the detector.

Lot of chemistry possible right up to KFO. But what can experiments see?

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Take a simple sub-problem

1935: Bethe explored the formation of deuteron through the fusion reaction $pn \to d\gamma$. Found a relation between elastic scattering and the reaction cross sections.

1966: Peebles applied Bethe's formalism to the synthesis of nuclei after the big bang (BBN). Gives a precise result for the amount of baryonic matter in the universe. Cross checked by entirely different means by the WMAP/Planck results.

Deuteron binding energy is 2.23 MeV, 0.1% of its mass. Must be formed in the last scattering in the fireball, otherwise will be broken up by the next scattering. Explores the KFO "surface"!

Chemistry after chemical freezeout

dN_d	$=$		differential density of d
		dN_p	differential density of p
	$\times$	dN_d	differential density of n
	$\times$	P_c	probability of collisions
	$\times$	P_r	probability of reaction
	$\times$	$C(pn; d)$	how the phase spaces of pnd are related

Gongshi sheng needs one definition:

dN_i	$=$		differential density of particle i
		f_i	phase space density of i
	$\times$	$d^3x_i d^3p_i$	phase space element of i

Simplifying assumptions

Classical kinetics used. When the phase space density of particles is much smaller than one particle per phase space volume of $\hbar^3$, quantum correlations between particles may be neglected. Full quantum kinetics known, and may be used if one wants.

Heinz et al, 1999.

Long range interactions between nucleons neglected:

$$P_c = V\delta^3(x_p - x_n)$$

Since the scattering length is much larger than the range of the potential, corrections are small. Easy to include range if needed: smear out the delta-function.

Simplified kinematics

The recoil γ has been neglected in the master equation. Within this approximation, one has

$$C(pn; d) = d^3x_d d^3p_d \delta^3(x_d - x_p)\delta^3(p_p + p_n - p_d).$$

The recoil particle in the final state can be included in the formalism. This will modify the momentum conserving delta function. But since the energy of the recoil just subtracts the binding energy, the difference is negligible.

In exact kinematics, when the proton's speed in the CM frame is in the range $\beta_p \ll 1$, then the deuteron's speed is $\beta_d = \beta_p^2/2$. As a result, the deuteron may be considered to be at rest in the CM frame.

The other factors

The most crucial input is

$$P_r = 1 - |S|^2 = \frac{k^2 \sigma_d}{4\pi},$$

where S is the S-matrix element for the scattering $pn \to pn$. If a deuteron is formed then $|S|^2 < 1$. Here, σ_d is the semi-inclusive cross section for the reaction $pn \to d+$anything.

Bethe, 1935

We can also write

$$f_i = \frac{1}{V}\rho_i, \quad \text{where} \quad \rho_i = \frac{d^3 N_i}{dp^3}.$$

Then, using the delta-functions to do various integrals, the master equation becomes

$$\rho_d(p_d) = \int d^3 k P_r \rho_p \left(\frac{p_d + k}{2}\right) \rho_n \left(\frac{p_d - k}{2}\right).$$

The coalescence model

The coalescence model is simply

$$P_r = B\delta^3(k).$$

All details of strong interactions are lumped into the parameter B. Then the master equation simplifies to

$$E_d \frac{d^3 N_d}{dp_d^3} = B\ E\frac{d^3 N_p}{dp^3}\ E\frac{d^3 N_n}{dp^3}, \qquad (2p = p_d).$$

Published data indicate that B corresponds to the length scale

$$\frac{1}{\sqrt{B}} \simeq 5\ \mathrm{fm}$$

Since this is within a factor of π of the range of nuclear interactions, one is motivated to explore the physics in more detail.

Is it possible to develop a dynamical nuclear physics model which predicts B?

Fusion dynamics

The scattering amplitude is given by

$$S = \frac{g(k^2) - ik}{g(k^2) + ik} \qquad \text{and} \qquad g(k^2) = -\frac{1}{a + ia''}\left(1 + \frac{1}{2}r^2k^2 + \cdots\right).$$

The elastic scattering length $a = -23.6$ fm, $r = 5.5$ fm and $a'' = 2.2$ fm. For $k < 60$ MeV, the $O(k^2)$ term can be neglected, so $P_r = 1 - |S|^2 \simeq 4ka''$

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Use thermal phase space densities

$$\rho_i(p, T) = \frac{N_i}{(2\pi M_i T)^{3/2}} \exp\left[-\frac{p^2}{2M_i T}\right],$$

where N_i is the total number of particles of type i with mass M_i.
N_p fixed by experiment: do not use feed-down corrected value.
Isospin chemical potential then fixes N_n.

LBN is not BBN

With all these inputs one finds

$$\rho_d \propto [M_d T(a'')^2]^3 \mathrm{e}^{-p^2/(2M_d T)} \int_0^{Ka''} dz\, z^3 \mathrm{e}^{-z^2/[4MT(a'')^2]}.$$

In BBN $T < 1$ MeV, so $\sqrt{2MT} < 40$ MeV. With an UV cutoff $K \simeq 60$ MeV, the approximation captures the physics fairly well. Bethe and Peebles got it right.

In HIC, $T \simeq 100$ MeV, so $\sqrt{2MT} \simeq 400$ MeV. Low momentum approximation does not work: integral goes as K^4! Why?

Although the probability of reactions is very small when relative momentum of np is large, the number of such pairs is much larger. Phase space trumps matrix element!

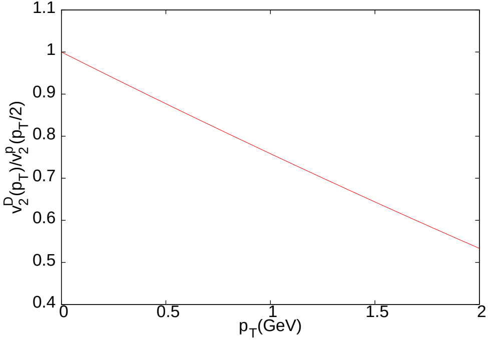
Coalescence is a bad approximation.

Smoking gun?

Coalescence implies constituent scaling of v_2, *i.e.*,

$$v_2^d(p_T) = 2v_2^p(p_T/2).$$

This fails when the momenta of the constituents are not equal. The dynamical theory of fusion predicts a specific p_T dependence of v_2^d/v_2^p when $\mu_I \simeq 0$. Also predict v_4^d/v_4^p.



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Main results

- Understanding freeze out is necessary to understand critical fluctuations.
- Single chemical freeze out is a good approximation for a subset of hadrons, and because the range of freezeout temperatures is not large.
- Good arguments that isospin freezes out only at the kinetic freezeout. Also perhaps resonances: K^*, ρ, N^* *etc.*. Observations difficult.
- Yield data gives evidence that strange and non-strange hadrons may have separate chemical freeze out. More complex explanations also possible.
- Light nuclei are synthesized only at the last scattering surface. Good physics reason to believe that coalescence does not work. The detailed model predicts the elliptic flow ratio for deuteron and proton.